Methods In Higher Forcing Axioms: Problems

v. 2, August 8, 2019

This is a list of problems that we are interested in discussing during the meeting. Feel free to send us additional problems by September 3, 2019. Any problem unnamed is proposed by the organisers.

We will select a few problems (in addition to the first problem on the list) and discuss them during the meeting. Please let us know if certain problems catches your fancy more than others, and we will try to make a democratic decision about the topics of discussion.

(1) Organising the existing knowledge on higher forcing axioms into categories and listing the main resources for each category, as well as the major results.

(2) The consistency of GCH together with the existence of a successor cardinal κ such that $\mathsf{SATP}(\kappa)$ and $\mathsf{SATP}(\kappa^+)$ both hold, where $\mathsf{SATP}(\lambda)$, for a given successor cardinal λ, is the statement "There is a λ-Aronszajn tree and all such trees are special". [This would answer a well-known question of Shelah. The hope is that one can combine the techniques from Aspero-Golshani's paper "The special Aronszajn tree property at $\aleph_2$ and GCH" with, for example, the proof from Abraham's paper "Aronszajn trees on $\aleph_2$ and $\aleph_3$".]

(3) Using side conditions to obtain models with universal (triangle-free, etc.) graphs of size $\aleph_1$ together with large continuum.

(4) Exploring consequences of (consistent) high forcing axioms (e.g. forcing axioms implying $\Box_{\omega_1}$). Concurrently to this, proving forcing axioms to be inconsistent due to their implying false statements (failures of ZFC-provable club-guessing, uniformization). (For example the forcing axiom, relative to collections of $\aleph_2$-many dense sets, for the class of proper forcing notions with the $\aleph_2$-c.c. is false; hopefully one can extend this type of result.)

(5) (**Miyamoto**) (CH) Find an interesting a class of posets Γ including the class of posets from Shelah's Generalized Martin's Axiom but also containing, for example, posets for adding suitable clubs of ω_2 with countable approximations, and for which the forcing axiom, with respect to $\aleph_2$-many dense sets, for Γ is consistent. It seems this should be approachable via a countable support iteration with side conditions consisting of $\aleph_1$-sized models. This forcing axiom should entail negations of tail club-guessing on S^2_1.

(6) (**Mota**) Does the bounded forcing axiom for the class of ω-proper forcings imply $2^{\aleph_0} = \aleph_2$? (All arguments for showing $2^{\aleph_0} = \aleph_2$ from the Bounded Proper Forcing Axiom involve some principle forcible by MRP-style forcings, which are not ω-proper. On the other hand, obtaining a model of the bounded forcing axiom for ω-proper forcings with a large continuum looks totally hopeless. This indicates that there should be a new family of coding techniques, beyond MRP, involving ω-proper forcing.)

(7) (**Abraham–Džamonja**) We would like to review the Abraham–Rubin–Shelah paper ON THE CONSISTENCY OF SOME PARTITION THEOREMS FOR CONTINUOUS COLORINGS, AND THE STRUCTURE OF $\aleph_1$-DENSE REAL ORDER TYPES (*Ann. Pure Appl. Logic* **29** no. 2, 1985, pp 123–206). During the workshop we want to reintroduce the results and see if the open problems can be tackled with new methods and generalized to uncountable cardinals. The goal within the time constraints, of course, is to set up new research directions, rather than finding a complete solution.

(8) (**Abraham**) There are combinatorial statements that were proved to be consistent by difficult forcing construction, but which can be shown to follow from PFA by much easier arguments. An example is the proof by Todorčević of the consistency that every finitely bounded coloring of the pairs of ω_1 has a polychromatic uncountable set. This also appears in the paper SOME RESULTS IN POLYCHROMATIC RAMSEY THEORY (*J. Symbolic Logic* **72** (2007), no. 3, 865–896.) by Abraham, Cummings, and Smyth. It is also proved in that paper that Martin's Axiom is not strong enough for this result. Now the question proposed is this: Find an axiom that is stronger than MA, does not required any assumption beyond ZFC for its consistency proof, and yet is strong enough to derive such combinatorial statements.

(9) (**Abraham**) The second question is also connected with polychromatic Ramsey theory. In that paper cited above it is proved as a consequence of MM (Martin Maximum) that any 2-bounded coloring of ω_2 has a closed polychromatic set of order type ω_1. It is not known if MM implies the existence of such a closed set of longer order types. I don't even know if there is one of order type $\omega_1 + 1$.